Elektriciteit tutorial

RC-hoogdoorlaatfilter

(complexe uitwerking)

W. Van Wichelen

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Wat gaan we in deze les leren?

- Opstellen van de transfertfunctie van een gegeven RC-filter
- Bepalen van de fasehoek
- Bepalen van de kantelfrequentie
- We maken gebruik van de complexe rekenkunde
- Simulatie met LTspice

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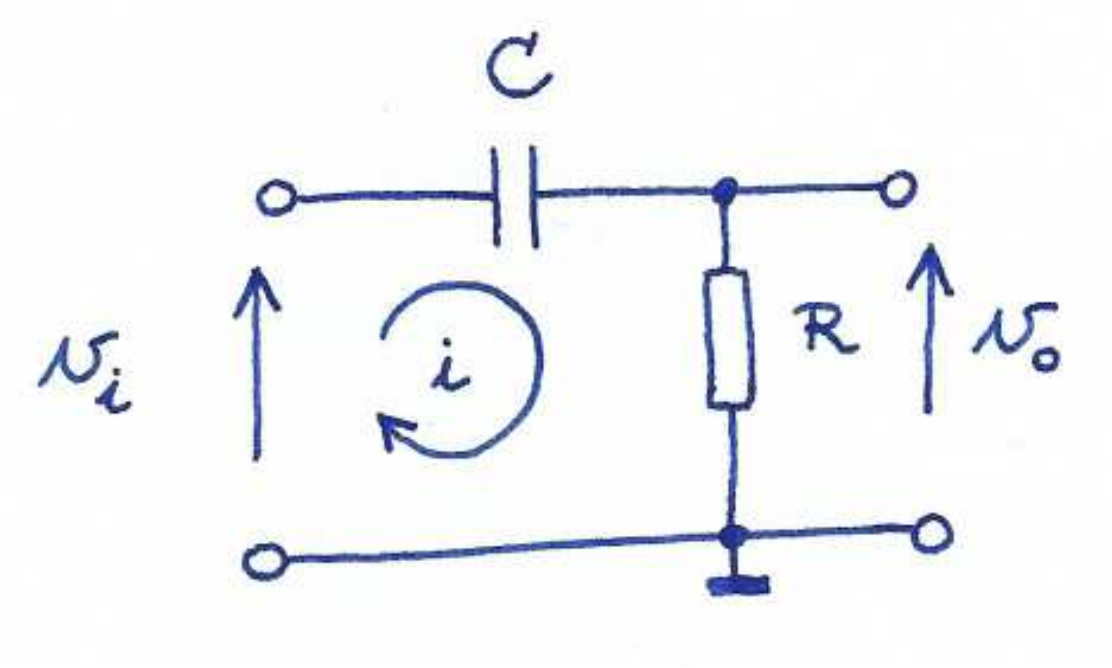
Schema



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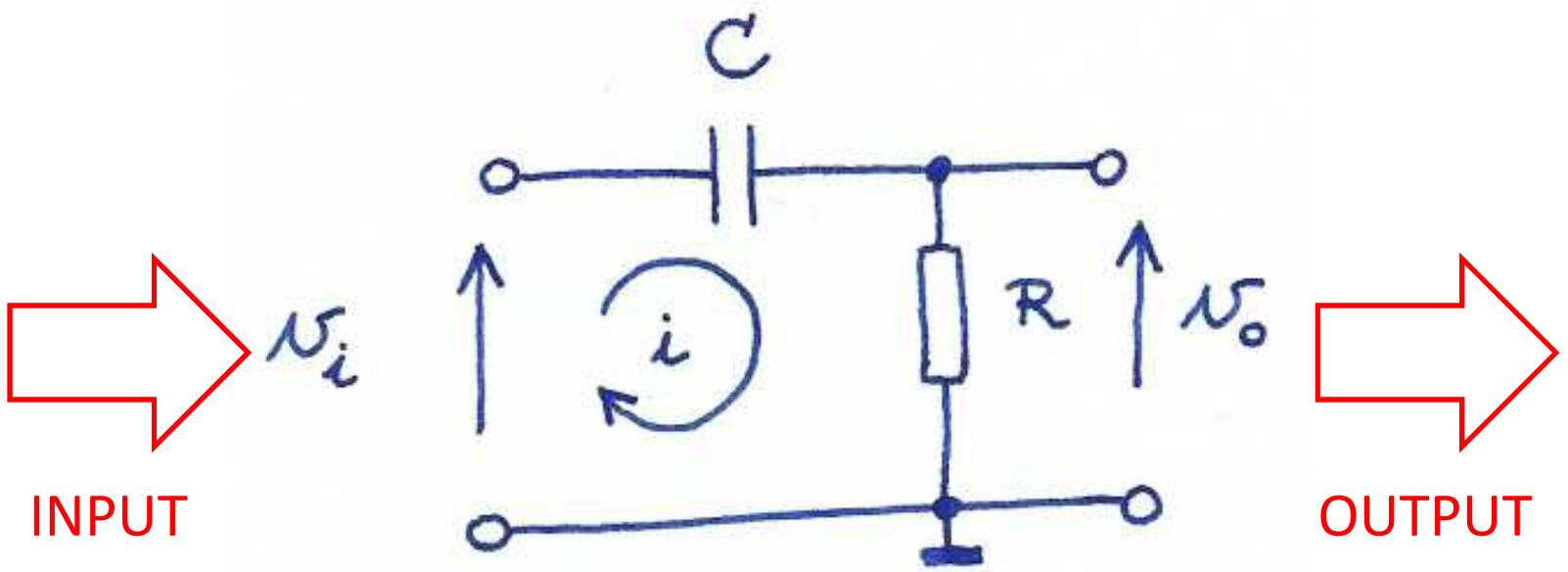
Principe



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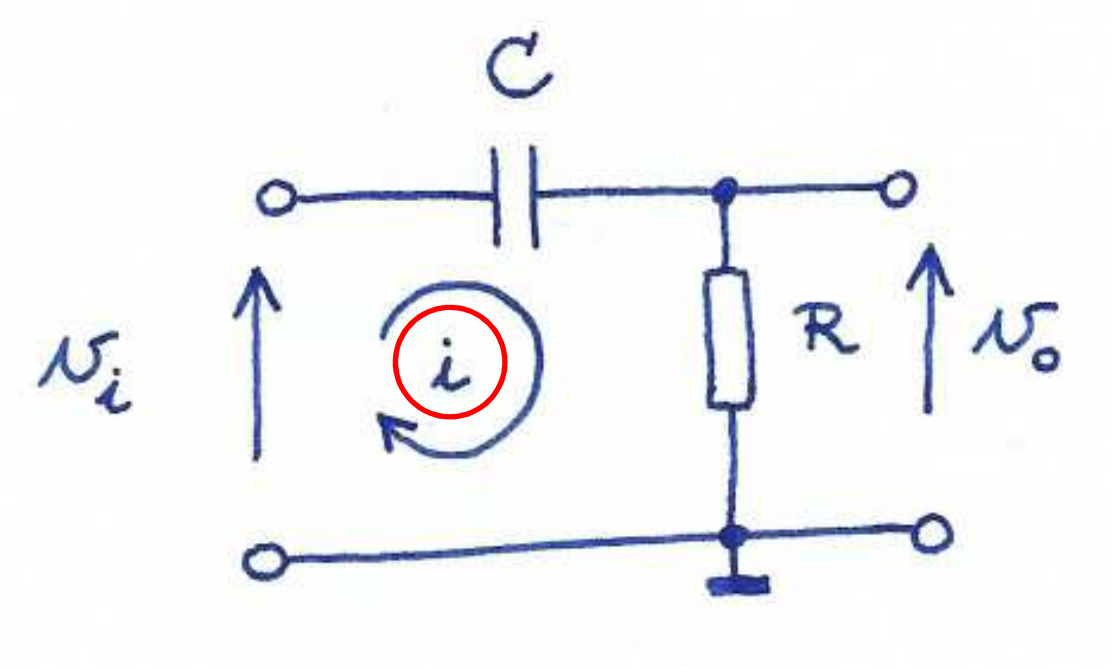
Stroom



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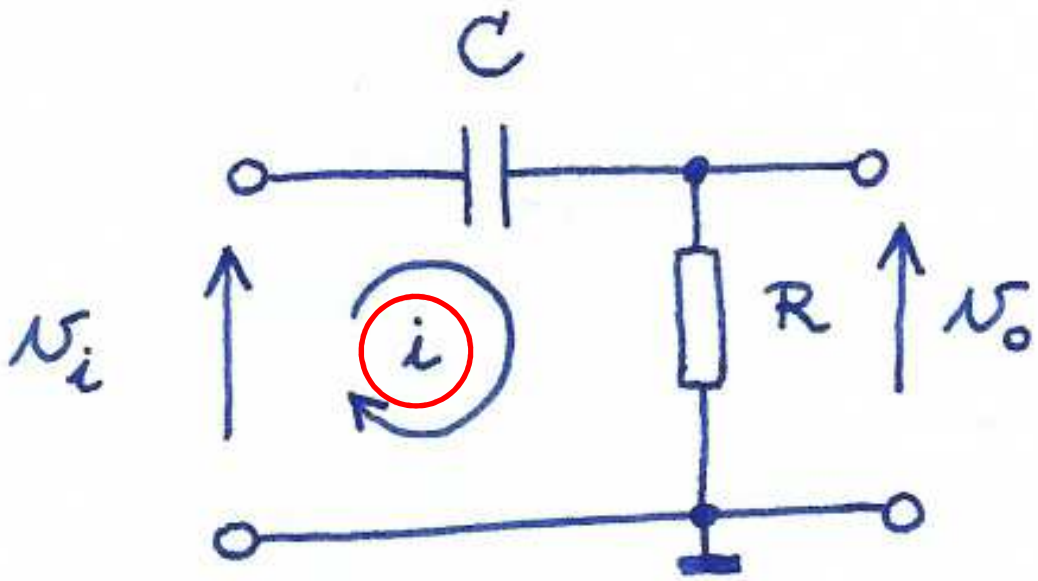
Stroom



$$i = \frac{v_i}{R + \frac{1}{j\omega C}} \qquad (1)$$

Complexe notatie voor Xc

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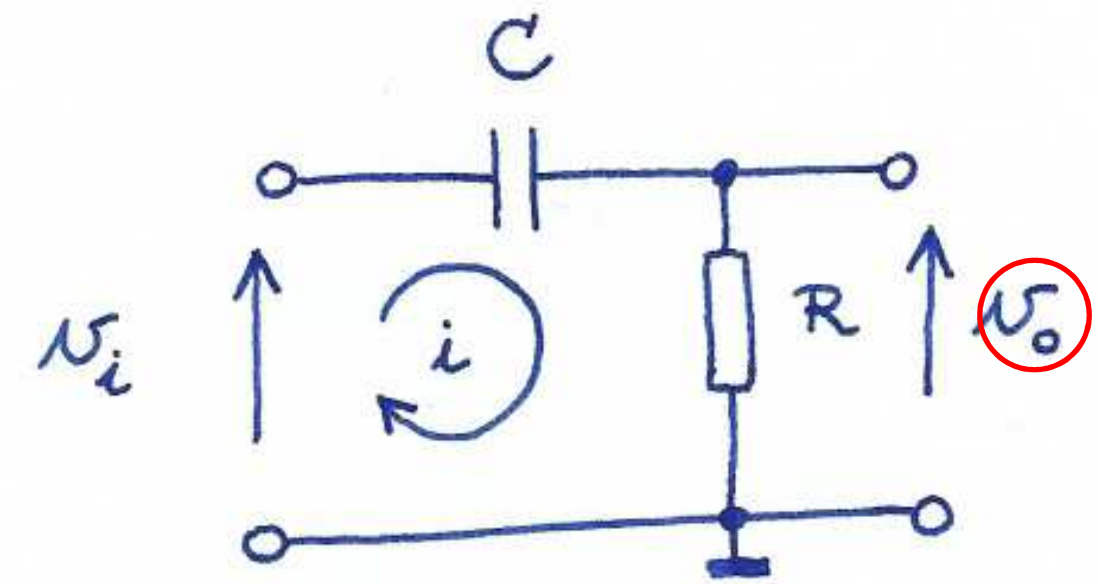
Uitgangsspanning



$$v_o = i \cdot R \qquad (2)$$

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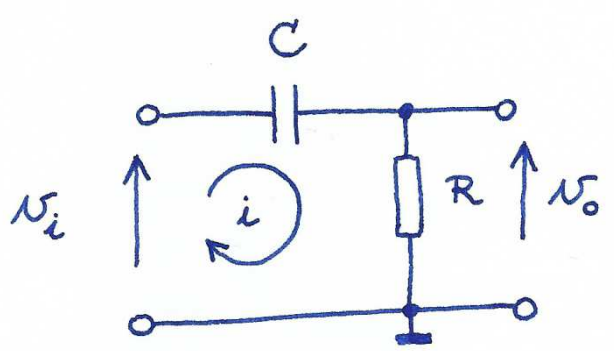
Uitgangsspanning



$$i = \frac{v_i}{R + \frac{1}{j\omega C}} \qquad (1)$$

$$v_o = i \cdot R \qquad (2)$$

(1) in (2) : substitutie

$$v_o = \frac{v_i}{R + \frac{1}{j\omega C}} \cdot R$$

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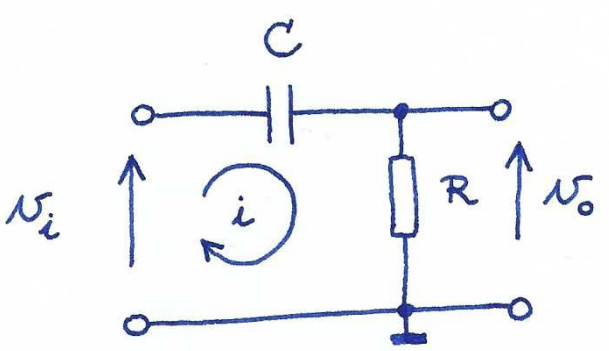
Spanningsversterking (Av)



$$v_o = \frac{v_i}{R + \frac{1}{j\omega C}} \cdot R$$

$$\Leftrightarrow \quad \frac{v_o}{v_i} = \frac{R}{R + \frac{1}{j\omega C}}$$

Deel linkerlid en rechterlid door v_i

'j' in de noemer!

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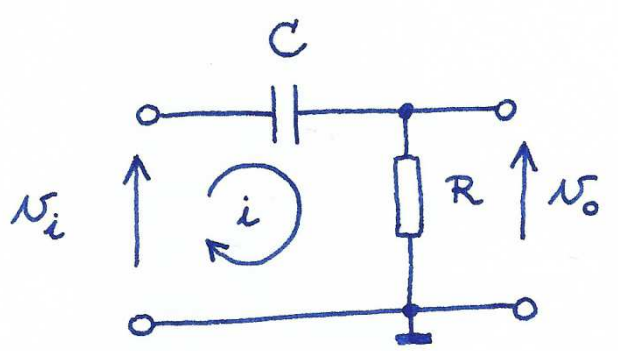
Spanningsversterking (Av)



$$\Leftrightarrow \quad \frac{V_o}{V_i} = \frac{R}{R + \frac{1}{j\omega C} \cdot j}$$

'j^2' = -1

$$\Leftrightarrow \quad \frac{V_o}{V_i} = \frac{R}{R - \frac{1}{\omega C} j}$$

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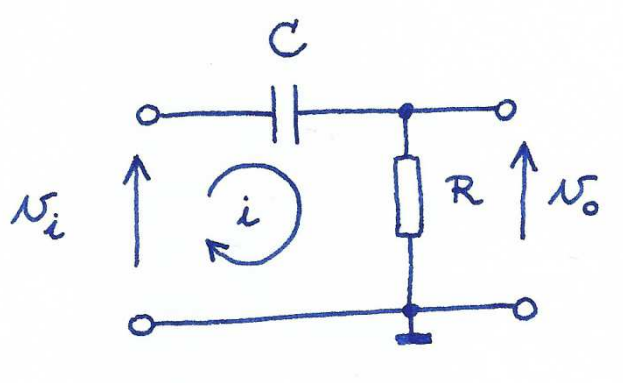
Spanningsversterking (Av)



$$\Leftrightarrow \quad \frac{v_o}{v_i} = \frac{R \;/R}{R/R - \frac{1}{\omega C}\, j/R}$$

$$\Leftrightarrow \quad \frac{v_o}{v_i} = \frac{1}{1 - \frac{1}{\omega RC}\, j}$$

Teller en noemer delen door ‘R’

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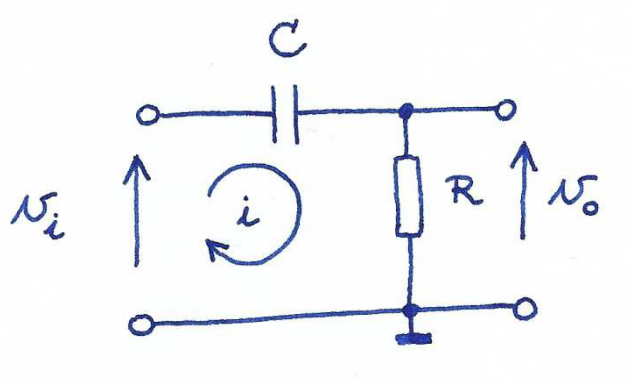
Spanningsversterking (Av)



$$\Leftrightarrow \quad \frac{v_o}{v_i} = \frac{1}{\underbrace{1}_{a} - \underbrace{\frac{1}{\omega RC}}_{b} j}$$

Teller en noemer
x complex toegevoegde

$$\Leftrightarrow \quad \frac{v_o}{v_i} = \frac{1 + \frac{1}{\omega RC} j}{\underbrace{1^2}_{a^2} + \underbrace{\left(\frac{1}{\omega RC}\right)^2}_{b^2}}$$

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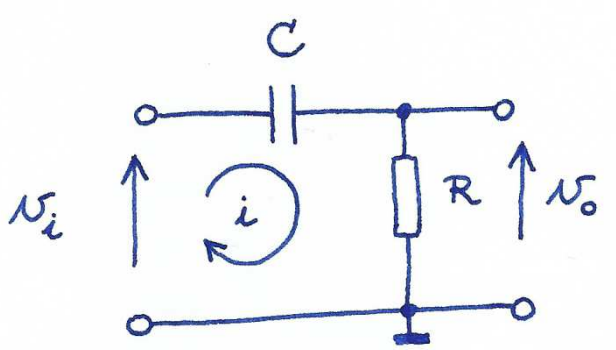
Spanningsversterking (Av)



$$\Leftrightarrow \quad \frac{V_o}{V_i} = \frac{1 + \frac{1}{\omega RC} j}{1^2 + \left(\frac{1}{\omega RC}\right)^2}$$

$$\Leftrightarrow \quad \frac{V_o}{V_i} = \frac{1 + \frac{1}{\omega RC} j}{1 + \frac{1}{(\omega RC)^2}}$$

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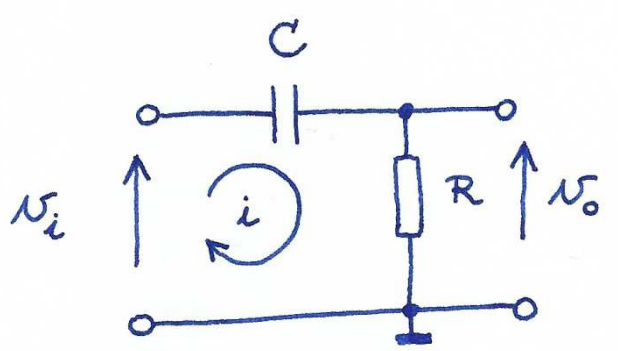
Spanningsversterking (Av)



$$\Leftrightarrow \quad \frac{V_o}{V_i} = \frac{1 + \frac{1}{\omega RC} j}{1 + \frac{1}{(\omega RC)^2}}$$

$$\frac{V_o}{V_i} = \frac{1}{1 + \frac{1}{(\omega RC)^2}} + \frac{\frac{1}{\omega RC} j}{1 + \frac{1}{(\omega RC)^2}}$$

Reële deel

Imaginaire deel

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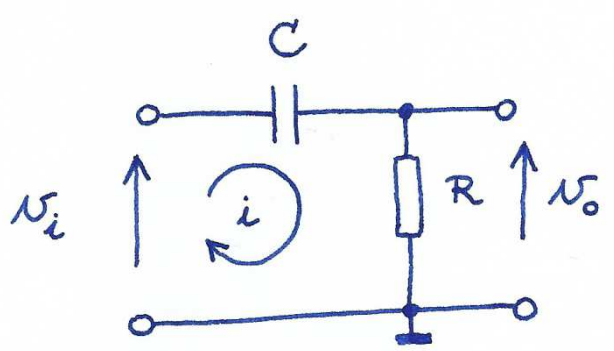
Modulus (grootte) Av



$$\frac{V_o}{V_i} = \frac{1}{1 + \frac{1}{(\omega RC)^2}} + \frac{\frac{1}{\omega RC} j}{1 + \frac{1}{(\omega RC)^2}}$$

$$\left| \frac{V_o}{V_i} \right| = \sqrt{Re^2 + Im^2}$$

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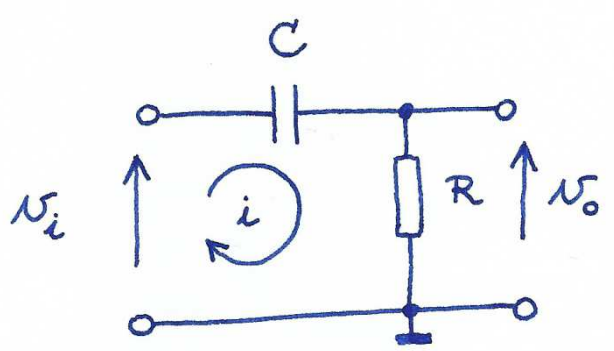
Modulus (grootte) Av



$$\left|\frac{V_o}{V_i}\right| = \sqrt{Re^2 + Im^2}$$

$$\rightarrow \left|\frac{V_o}{V_i}\right| = \sqrt{\left(\frac{1}{1+\frac{1}{(\omega RC)^2}}\right)^2 + \left(\frac{\frac{1}{\omega RC}}{1+\frac{1}{(\omega RC)^2}}\right)^2}$$

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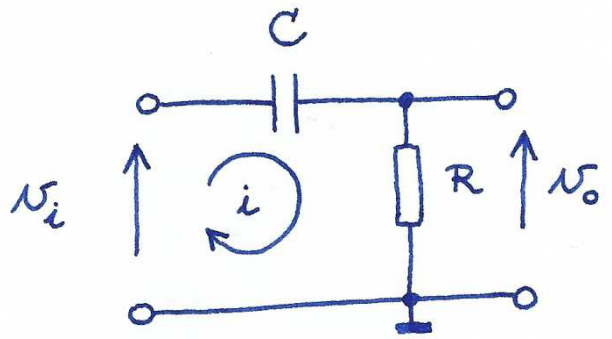
Modulus (grootte) Av



$$\rightarrow \left|\frac{V_o}{V_i}\right| = \sqrt{\left(\frac{1}{1+\frac{1}{(\omega RC)^2}}\right)^2 + \left(\frac{\frac{1}{\omega RC}}{1+\frac{1}{(\omega RC)^2}}\right)^2}$$

$$= \sqrt{\frac{1}{\left(1+\frac{1}{(\omega RC)^2}\right)^2} + \frac{\left(\frac{1}{\omega RC}\right)^2}{\left(1+\frac{1}{(\omega RC)^2}\right)^2}}$$

Op dezelfde noemer brengen

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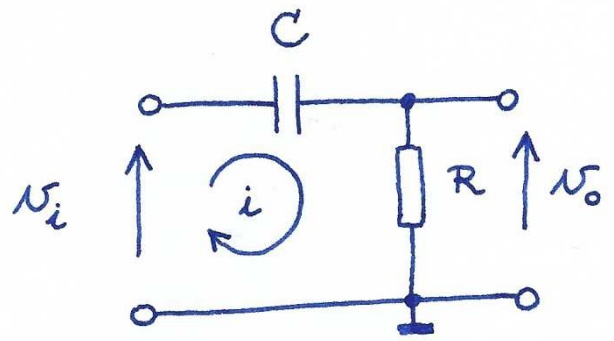
Modulus (grootte) Av



$$= \sqrt{\frac{1}{\left(1+\frac{1}{(\omega RC)^2}\right)^2} + \frac{\left(\frac{1}{\omega RC}\right)^2}{\left(1+\frac{1}{(\omega RC)^2}\right)^2}}$$

$$= \sqrt{\frac{1+\frac{1}{(\omega RC)^2}}{\left(1+\frac{1}{(\omega RC)^2}\right)^2}}$$

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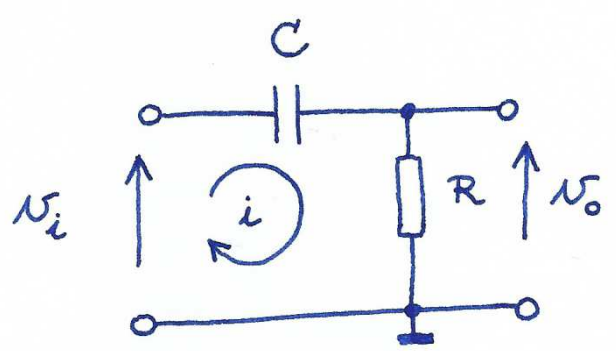
Modulus (grootte) Av



$$= \sqrt{\frac{1 + \frac{1}{(\omega RC)^2}}{\left(1 + \frac{1}{(\omega RC)^2}\right)^2}}$$

A

A^2

van de vorm:
$A / A^2 = 1 / A$

$$= \sqrt{\frac{1}{1 + \frac{1}{(\omega RC)^2}}}$$

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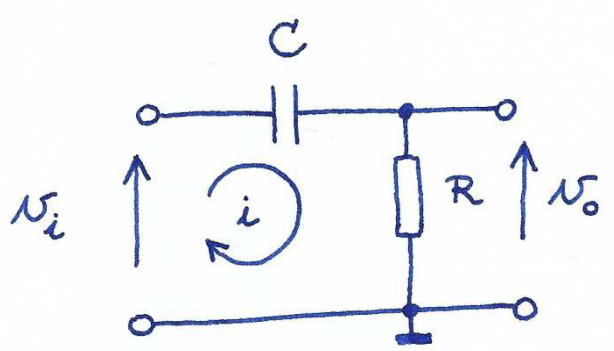
Modulus (grootte) Av



$$= \sqrt{\frac{1}{1 + \frac{1}{(\omega RC)^2}}}$$

$$= \frac{1}{\sqrt{1 + \frac{1}{(\omega RC)^2}}}$$

VKW van de teller /
VKW van de noemer

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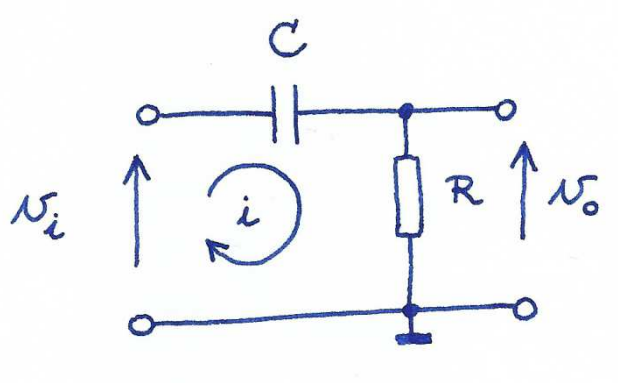
Modulus (grootte) Av



$$= \frac{1}{\sqrt{1 + \frac{1}{(\omega RC)^2}}}$$

Finale uitdrukking voor de grootte van de spanningsversterking Av

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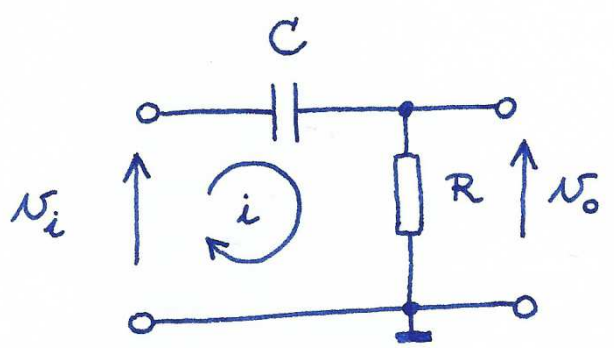
Fasehoek φ



$$\frac{V_o}{V_i} = \frac{1}{1 + \frac{1}{(\omega RC)^2}} + \frac{\frac{1}{\omega RC} j}{1 + \frac{1}{(\omega RC)^2}}$$

$$\varphi = \mathrm{Bgtg}\left(\frac{Im}{Re}\right)$$

$$\rightarrow \quad \varphi = \mathrm{Bgtg}\left(\frac{\frac{\frac{1}{\omega RC}}{1 + \frac{1}{(\omega RC)^2}}}{\frac{1}{1 + \frac{1}{(\omega RC)^2}}}\right)$$

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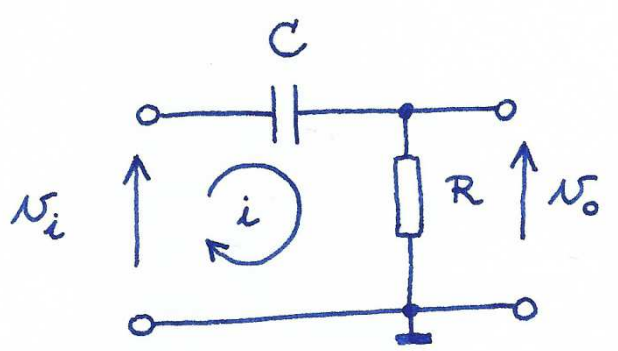
Fasehoek φ



$$\rightarrow \quad \varphi = \text{Bgtg}\left(\frac{\frac{1}{\omega RC}}{1+\frac{1}{(\omega RC)^2}} \Big/ \frac{1}{1+\frac{1}{(\omega RC)^2}}\right)$$

$$\Leftrightarrow \quad \varphi = \text{Bgtg}\,\frac{1}{\omega RC}$$

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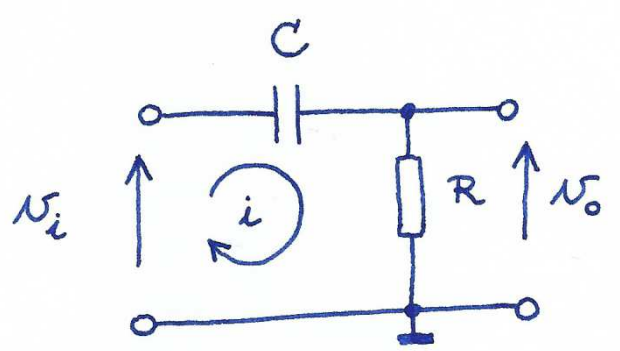
Fasehoek φ



$$\rightarrow \quad \varphi = \text{Bgtg}\left(\frac{\frac{1}{\omega RC}}{1+\frac{1}{(\omega RC)^2}} \Big/ \frac{1}{1+\frac{1}{(\omega RC)^2}}\right)$$

$$\Leftrightarrow \quad \boxed{\varphi = \text{Bgtg}\,\frac{1}{\omega RC}}$$

Finale uitdrukking voor de fasehoek tussen Vin en Vout

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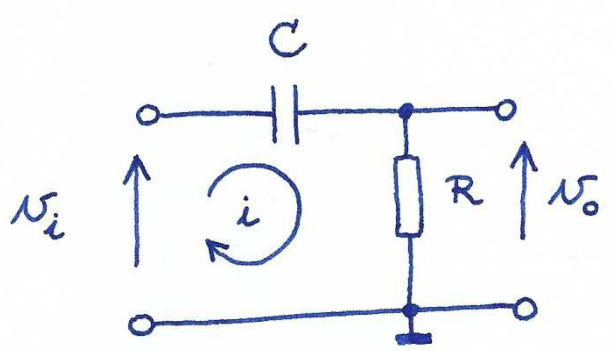
Kantelfrequentie f_k



$$f_k = ? \rightarrow \text{Stel} \left| \frac{v_o}{v_i} \right| = \frac{1}{\sqrt{2}}$$

Bepaal de frequentie waarbij 70% van de ingangsspanning (V_{in}) aan de uitgang (V_{out}) verschijnt.

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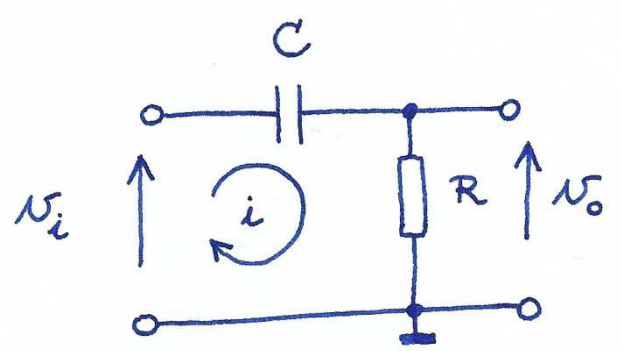
Kantelfrequentie f_k



$$f_k = ? \rightarrow \text{Stel} \left|\frac{v_o}{v_i}\right| = \frac{1}{\sqrt{2}}$$

$$\rightarrow \frac{1}{\sqrt{1 + \frac{1}{(\omega RC)^2}}} = \frac{1}{\sqrt{2}}$$

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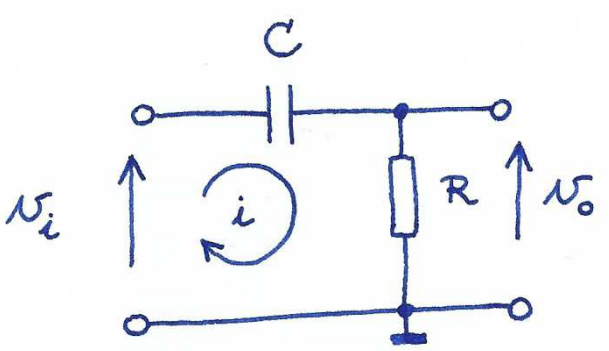
Kantelfrequentie f_k



$$\rightarrow \frac{1}{\sqrt{1+\frac{1}{(\omega RC)^2}}} = \frac{1}{\sqrt{2}}$$

$$\Leftrightarrow \left[\sqrt{1+\frac{1}{(\omega RC)^2}}\right]^2 = \left[\sqrt{2}\right]^2$$

$$\Leftrightarrow 1+\frac{1}{(\omega RC)^2} = 2$$

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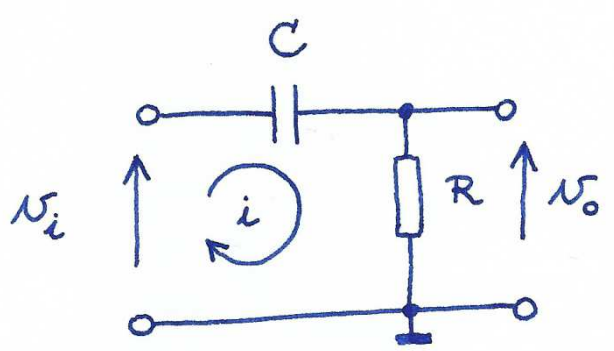
Kantelfrequentie f_k



$$\Leftrightarrow \quad 1 + \frac{1}{(\omega RC)^2} = 2$$

$$\Leftrightarrow \quad \frac{1}{(\omega RC)^2} = 2 - 1$$

$$\Leftrightarrow \quad \frac{1}{\omega^2 (RC)^2} = 1 \quad \Leftrightarrow \quad \omega^2 = \frac{1}{(RC)^2} \quad \Leftrightarrow \quad \omega_k = \sqrt{\frac{1}{(RC)^2}}$$

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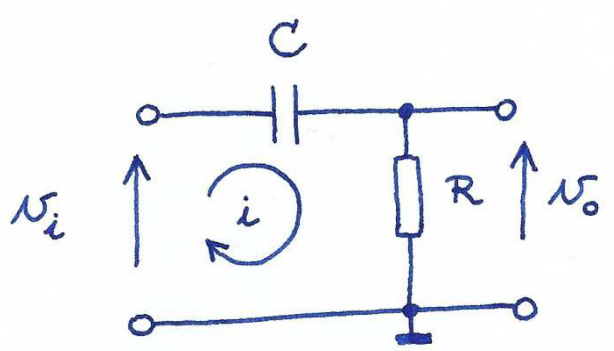
Kantelfrequentie f_k



$$\Leftrightarrow \frac{1}{\omega^2 (RC)^2} = 1 \quad \Leftrightarrow \quad \omega^2 = \frac{1}{(RC)^2} \quad \Leftrightarrow \quad \omega_k = \sqrt{\frac{1}{(RC)^2}}$$

$$\Leftrightarrow \omega_k = \frac{1}{RC} \quad \Leftrightarrow \quad 2\pi f_k = \frac{1}{RC} \quad \Leftrightarrow \quad \boxed{f_k = \frac{1}{2\pi RC}}$$

$\omega = 2 \,.\, \pi \,.\, f$

Finale uitdrukking
voor de kantelfrequentie f_k

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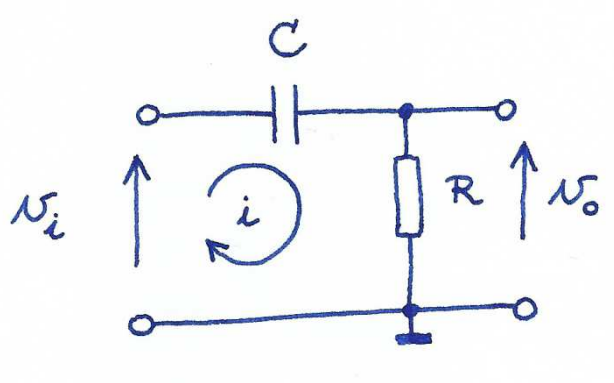
Tijdconstante τ



$$f_k = \frac{1}{2\pi RC}$$

Stel R . C = τ
(tijdconstante)

Alternatieve notatie:

$$f_k = \frac{1}{2 \cdot \pi \cdot \tau}$$

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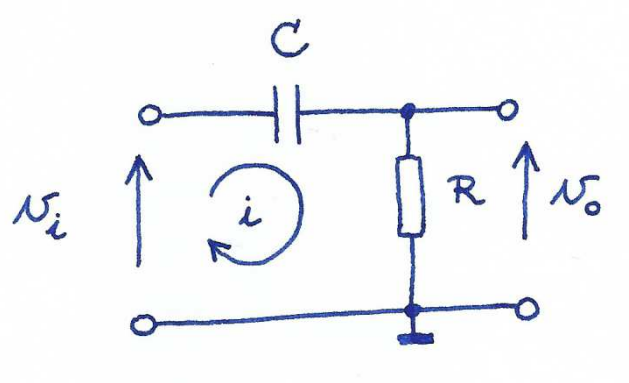
Voorbeeld



R = 330 Ω | C = 470 nF

$$f_k = \frac{1}{2 * \pi * R * C} = \frac{1}{2 * \pi * 330 * 470 * 10^{-9}}$$

$$f_k = 1026Hz$$

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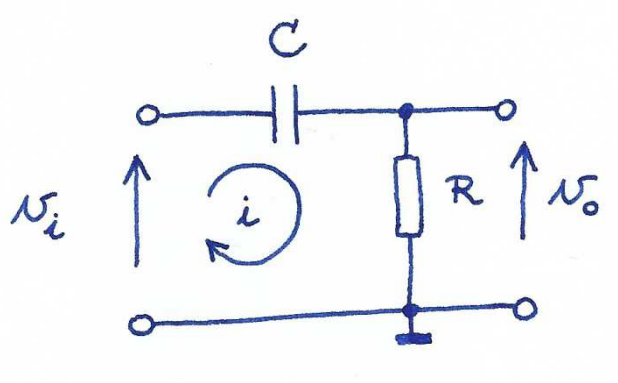
Voorbeeld



R = 330 Ω | C = 470 nF | f = 100 Hz

$$A_v = \frac{1}{\sqrt{1 + \frac{1}{(2 * \pi * f * R * C)^2}}}$$

$$A_v = \frac{1}{\sqrt{1 + \frac{1}{(2 * \pi * 100 * 330 * 470 * 10^{-9})^2}}}$$

$$A_v = 0{,}097 \rightarrow 9{,}7\%$$

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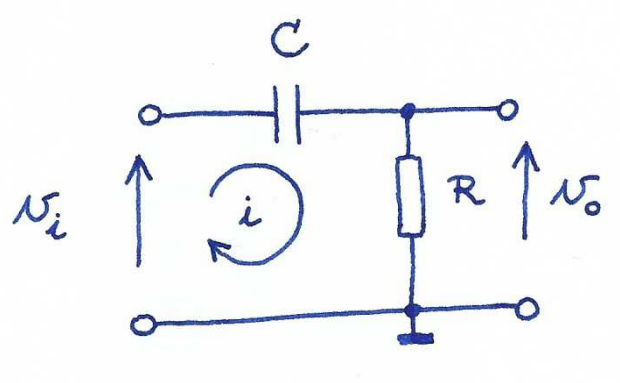
Voorbeeld



R = 330 Ω | C = 470 nF | f = 10 000 Hz

$$A_v = \frac{1}{\sqrt{1 + \frac{1}{(2 * \pi * f * R * C)^2}}}$$

$$A_v = \frac{1}{\sqrt{1 + \frac{1}{(2 * \pi * 10\,000 * 330 * 470 * 10^{-9})^2}}}$$

$$A_v = 0{,}995 \rightarrow 99{,}5\%$$

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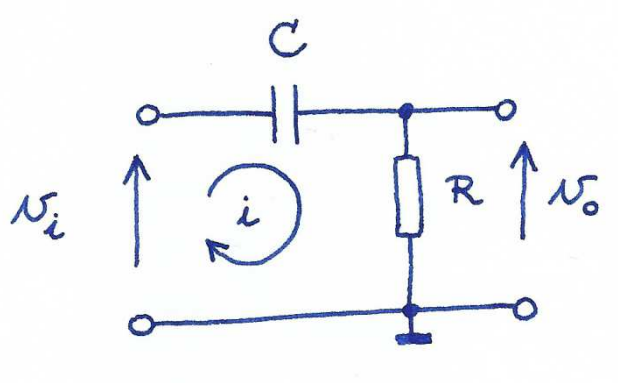
Frequentie-onderzoek



Voor een set van frequenties bepalen we:

- de spanningsversterking: A_V
- de fasehoek tussen IN en OUT: φ
- Dit resulteert in een AMPLITUDEKARAKTERISTIEK en een FASEKARAKTERISTIEK

HOE?

- rekenblad (Excel)
- Simulator LTspice

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EINDE

CREATIE & VOICE-OVER

W. Van Wichelen

DATUM SCREENCAST

2021.01.15

DOELPUBLIEK

Industriële ICT

GEBRUIKTE SOFTWARE

iSpring Free Cam

DATUM PUBLICATIE

2021.01.18

LEERPLANDOELEN

OO-2017-005/41/42/46

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