Elektriciteit tutorial

RC-laagdoorlaatfilter

(complexe uitwerking)

W. Van Wichelen

ELECTRIVIAL TUTORIALS

Wat gaan we in deze les leren?

- Opstellen van de transfertfunctie van een gegeven RC-filter
- Bepalen van de fasehoek
- Bepalen van de kantelfrequentie
- We maken gebruik van de complexe rekenkunde

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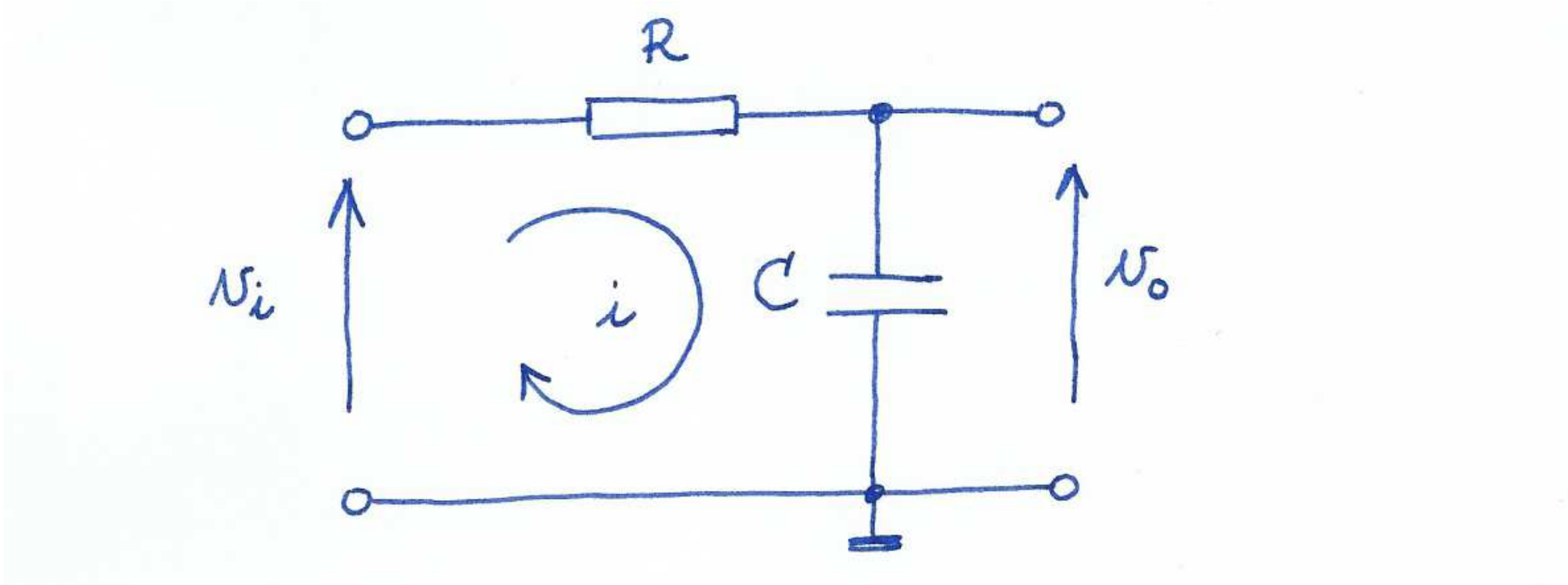
Schema



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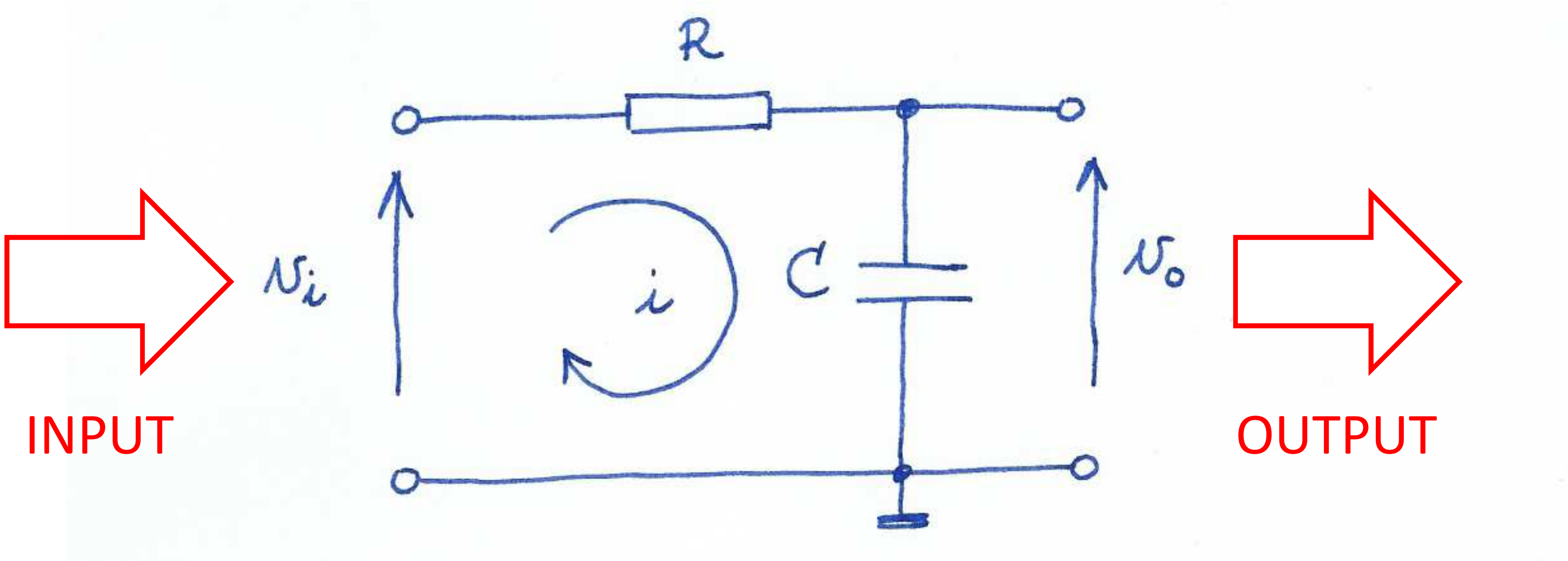
Principe



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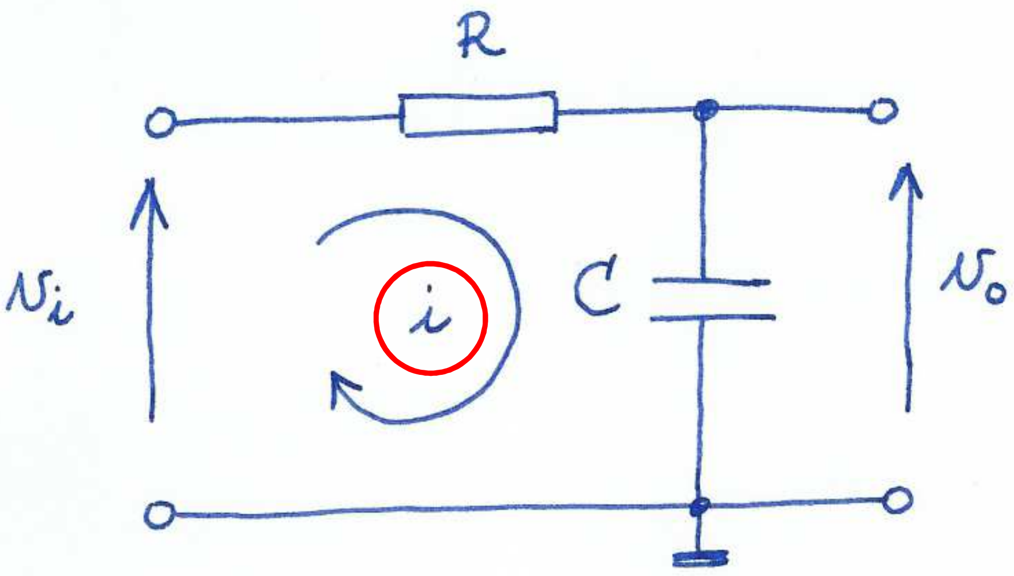
Stroom



$$i = \frac{v_i}{R + \frac{1}{j\omega C}} \qquad (1)$$

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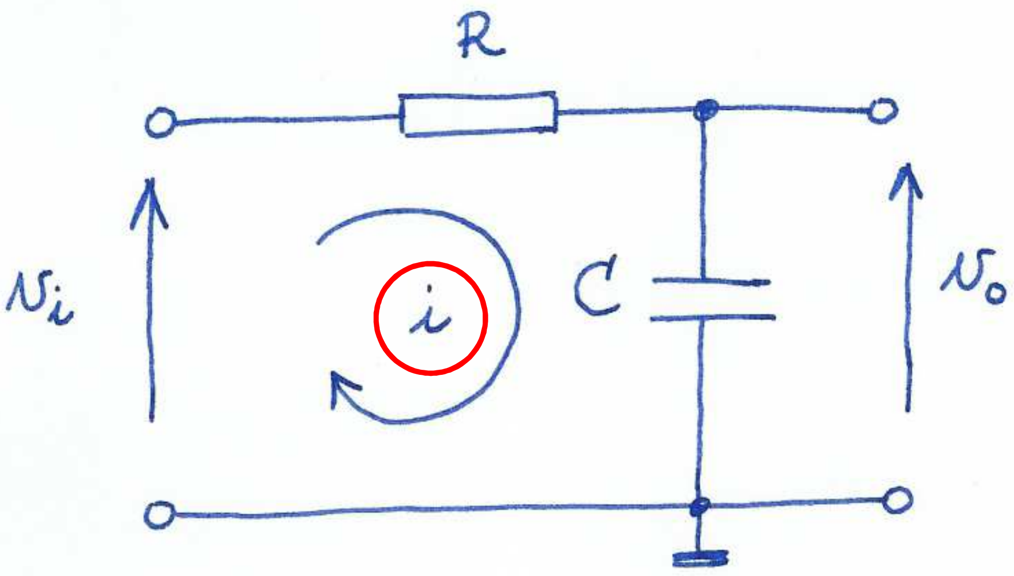
Stroom

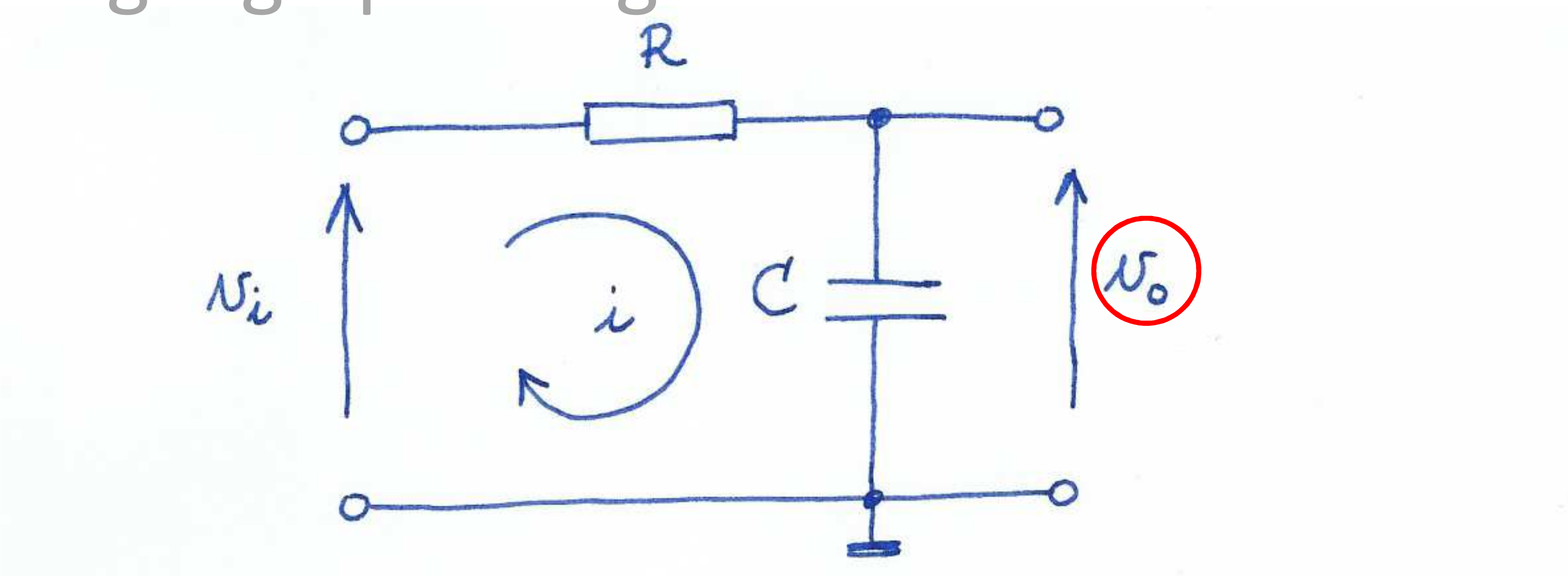


$$i = \frac{v_i}{R + \frac{1}{j\omega C}} \qquad (1)$$

Complexe notatie voor Xc

RC-LAAGDOORLAATFILTER

Uitgangsspanning

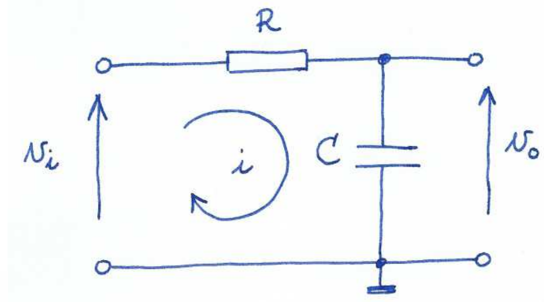


$$v_o = i \cdot \frac{1}{j\omega C} \qquad (2)$$

Complexe notatie voor Xc

RC-LAAGDOORLAATFILTER

Uitgangsspanning



$$i = \frac{v_i}{R + \frac{1}{j\omega C}} \qquad (1)$$

$$v_o = i \cdot \frac{1}{j\omega C} \qquad (2)$$

(1) in (2) : substitutie

$$v_o = \frac{v_i}{R + \frac{1}{j\omega C}} \cdot \frac{1}{j\omega C}$$

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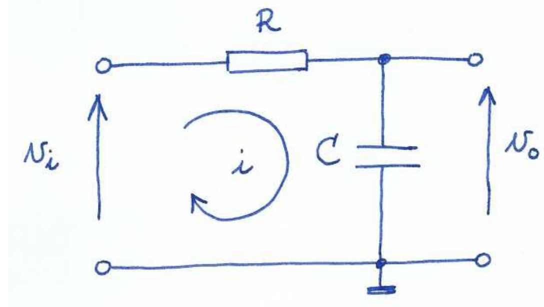
Spanningsversterking (A_v)



$$v_o = \frac{v_i}{R + \frac{1}{j\omega C}} \cdot \frac{1}{j\omega C}$$

Deel linkerlid en rechterlid door v_i

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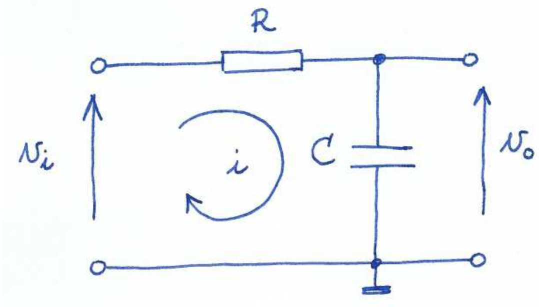
Spanningsversterking (Av)



$$v_o = \frac{v_i}{\left(R + \frac{1}{j\omega C}\right)} \cdot \frac{1}{j\omega C}$$

$$\Leftrightarrow \quad \frac{v_o}{v_i} = \frac{1}{j\omega RC + 1}$$

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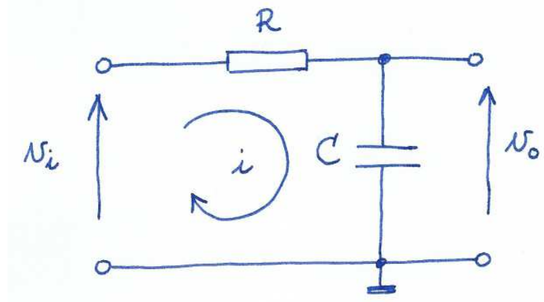
Spanningsversterking (Av)



$$v_o = \frac{v_i}{\left(R + \frac{1}{j\omega C}\right)} \cdot \frac{1}{j\omega C}$$

$$\Leftrightarrow \frac{v_o}{v_i} = \frac{1}{j\omega RC + 1}$$

Spanningsversterking Av

TRANSFERTFUNCTIE = OUTPUT/INPUT

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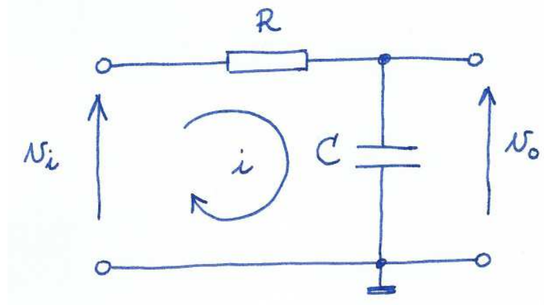
Spanningsversterking (A_v)



$$\frac{v_o}{v_i} = \frac{1}{j\omega RC \quad + \quad 1}$$

We herschikken de noemer

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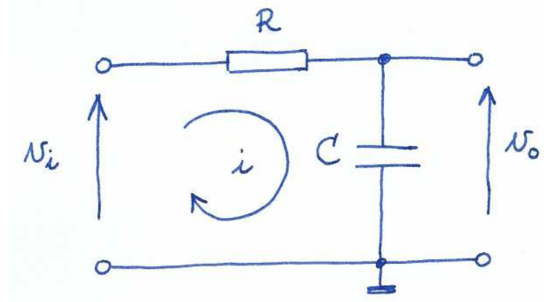
Spanningsversterking (Av)



$$\frac{v_o}{v_i} = \frac{1}{j\omega RC + 1}$$

$$\Leftrightarrow \quad \frac{v_o}{v_i} = \frac{1}{1 + \omega RC j}$$

‘j’ uit de noemer!

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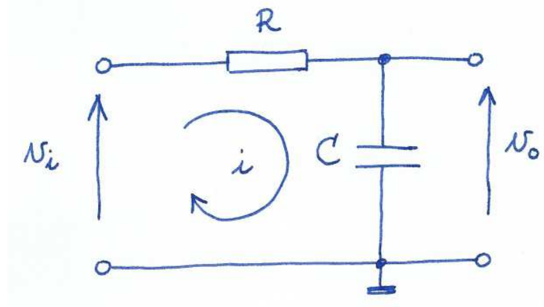
Spanningsversterking (Av)



$$\frac{v_o}{v_i} = \frac{1}{j\omega RC + 1}$$

$$\Leftrightarrow \quad \frac{v_o}{v_i} = \frac{1}{1 + \omega RC j}$$

Reëel deel — Imaginair deel

Teller en noemer vermenigvuldigen met het 'complex toegevoegde':
$(a + bj) \,.\, (a - bj) = a^2 + b^2$

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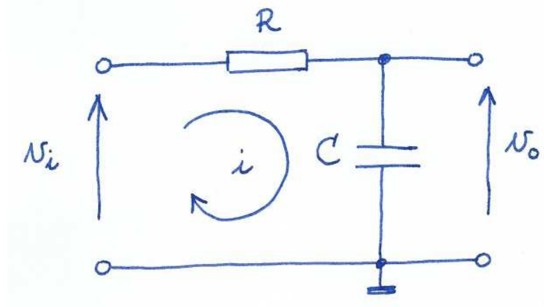
Spanningsversterking (Av)



$$\Leftrightarrow \quad \frac{v_o}{v_i} = \frac{1}{1 + \omega RCj}$$

$$\Leftrightarrow \quad \frac{v_o}{v_i} = \frac{1 - \omega RCj}{1^2 + (\omega RC)^2}$$

a - bj

a²   b²

'j' is verdwenen uit de noemer!

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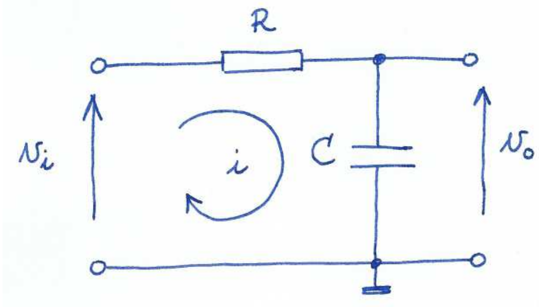
Spanningsversterking (Av)



$$\Leftrightarrow \quad \frac{v_o}{v_i} = \frac{1}{1 + \omega RCj}$$

$$\Leftrightarrow \quad \frac{v_o}{v_i} = \frac{1 - \omega RCj}{1^2 + (\omega RC)^2}$$

herschikken!

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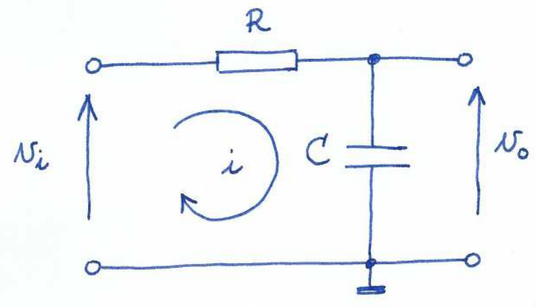
Spanningsversterking (Av)



$$\Leftrightarrow \frac{v_o}{v_i} = \frac{1}{1 + \omega RCj}$$

$$\Leftrightarrow \frac{v_o}{v_i} = \frac{1 - \omega RCj}{1^2 + (\omega RC)^2}$$

$$\Leftrightarrow \frac{v_o}{v_i} = \frac{1}{1+(\omega RC)^2} - \frac{\omega RC}{1+(\omega RC)^2} j$$

Reëel deel Imaginair deel

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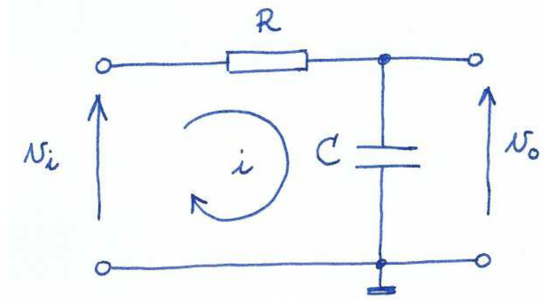
Modulus (grootte) Av



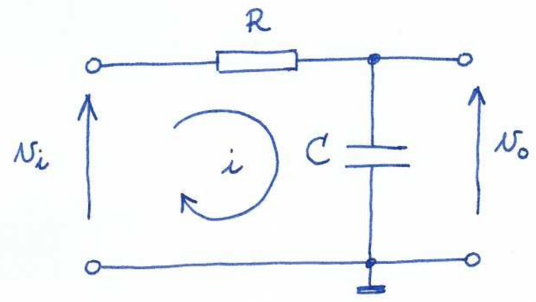
$$\Leftrightarrow \quad \frac{v_o}{v_i} = \frac{1}{1+(\omega RC)^2} - \frac{\omega RC}{1+(\omega RC)^2}\, j$$

$$\left|\frac{v_o}{v_i}\right| = \sqrt{Re^2 + Im^2}$$

$$\Leftrightarrow \quad \left|\frac{v_o}{v_i}\right| = \sqrt{\frac{1^2}{\left[1+(\omega RC)^2\right]^2} + \frac{(\omega RC)^2}{\left[1+(\omega RC)^2\right]^2}}$$

RC-LAAGDOORLAATFILTER

Modulus (grootte) Av



$$\Leftrightarrow \quad \frac{v_o}{v_i} = \frac{1}{1+(\omega RC)^2} - \frac{\omega RC}{1+(\omega RC)^2} j$$

$$\left|\frac{v_o}{v_i}\right| = \sqrt{Re^2 + Im^2}$$

$$\Leftrightarrow \quad \left|\frac{v_o}{v_i}\right| = \sqrt{\frac{1^2}{\left[1+(\omega RC)^2\right]^2} + \frac{(\omega RC)^2}{\left[1+(\omega RC)^2\right]^2}}$$

Op één noemer brengen!

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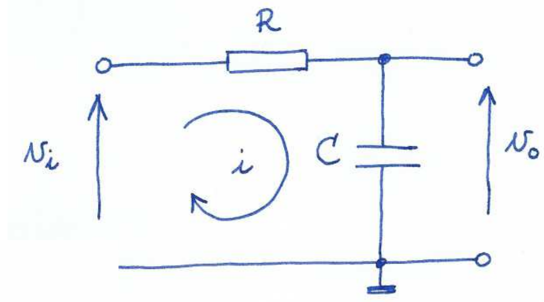
Modulus (grootte) Av



$$\left|\frac{v_o}{v_i}\right| = \sqrt{Re^2 + Im^2}$$

$$\Leftrightarrow \left|\frac{v_o}{v_i}\right| = \sqrt{\frac{1^2}{\left[1+(\omega RC)^2\right]^2} + \frac{(\omega RC)^2}{\left[1+(\omega RC)^2\right]^2}}$$

$$\Leftrightarrow \left|\frac{v_o}{v_i}\right| = \sqrt{\frac{1+(\omega RC)^2}{\left[1+(\omega RC)^2\right]^2}}$$

A

A²

van de vorm:

$A / A^2 = 1 / A$

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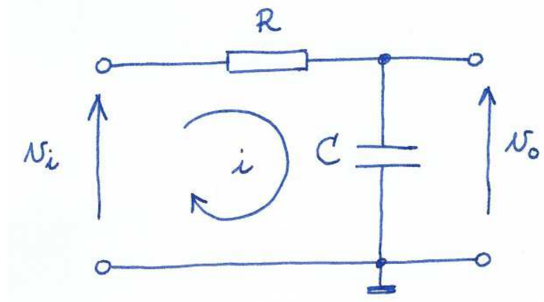
Modulus (grootte) Av



$$\Leftrightarrow \left|\frac{v_o}{v_i}\right| = \sqrt{\frac{1+(\omega RC)^2}{\left[1+(\omega RC)^2\right]^2}}$$

$$\Leftrightarrow \left|\frac{v_o}{v_i}\right| = \sqrt{\frac{1}{1+(\omega RC)^2}}$$

VKW van de teller /
VKW van de noemer

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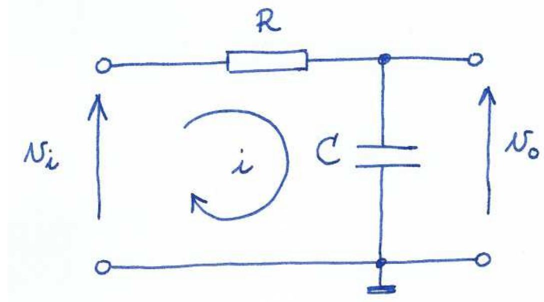
Modulus (grootte) Av



$$\Leftrightarrow \left|\frac{v_o}{v_i}\right| = \sqrt{\frac{1}{1+(\omega RC)^2}}$$

$$\Leftrightarrow \left|\frac{v_o}{v_i}\right| = \frac{1}{\sqrt{1+(\omega RC)^2}}$$

Finale uitdrukking voor de grootte van de spanningsversterking Av

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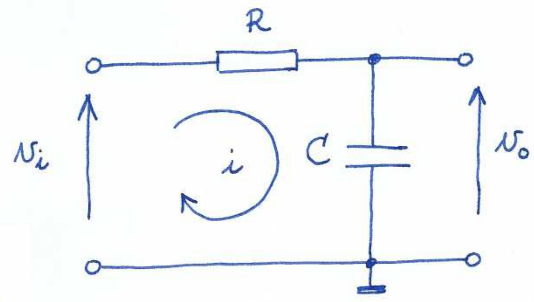
Fasehoek φ



$$\frac{v_o}{v_i} = \frac{1}{1+(\omega RC)^2} - \frac{\omega RC}{1+(\omega RC)^2}\, j$$

$$\varphi = \text{Bgtg}\left(\frac{Im}{Re}\right)$$

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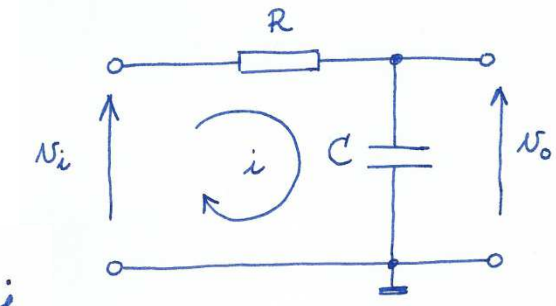
Fasehoek φ



$$\frac{v_o}{v_i} = \frac{1}{1+(\omega RC)^2} - \frac{\omega RC}{1+(\omega RC)^2}\,j$$

$$\varphi = \mathrm{Bgtg}\left(\frac{Im}{Re}\right)$$

$$\Rightarrow \quad \varphi = \mathrm{Bgtg}\left(\frac{-\dfrac{\omega RC}{1+(\omega RC)^2}}{\dfrac{1}{1+(\omega RC)^2}}\right)$$

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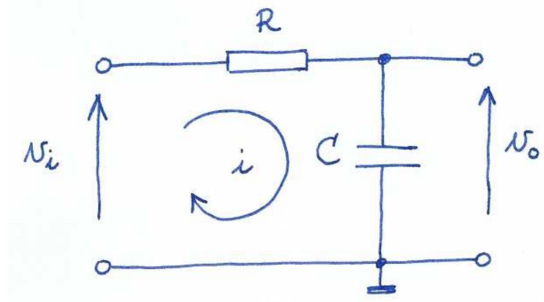
Fasehoek φ



$$\varphi = \text{Bgtg}\left(\frac{Im}{Re}\right)$$

$$\Rightarrow \quad \varphi = \text{Bgtg}\left(\frac{-\dfrac{\omega RC}{\cancel{1+(\omega RC)^2}}}{\dfrac{1}{\cancel{1+(\omega RC)^2}}}\right)$$

Teller en noemer vermenigvuldigen met ‘1 + (ωRC)²’

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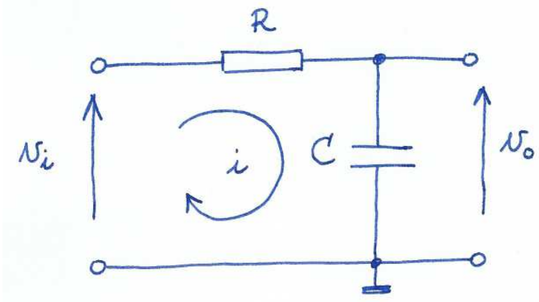
Fasehoek φ



$$\varphi = \text{Bgtg}\left(\frac{Im}{Re}\right)$$

$$\Rightarrow \quad \varphi = \text{Bgtg}\left(\frac{-\dfrac{\omega RC}{1+(\omega RC)^2}}{\dfrac{1}{1+(\omega RC)^2}}\right)$$

$$\Leftrightarrow \quad \boxed{\varphi = \text{Bgtg}(-\omega RC)}$$

Finale uitdrukking voor de fasehoek tussen v_{in} en v_{out}

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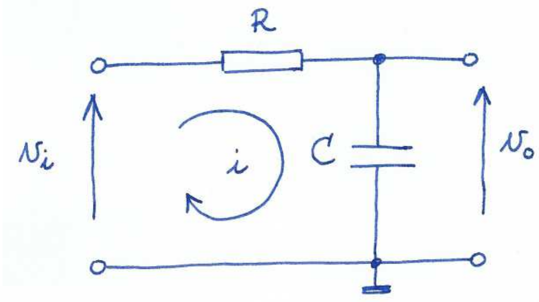
Kantelfrequentie f_k



Kantelfrequentie : f_k

Stel $\left|\frac{v_o}{v_i}\right| = \frac{1}{\sqrt{2}} \approx 0,7$ (70%)

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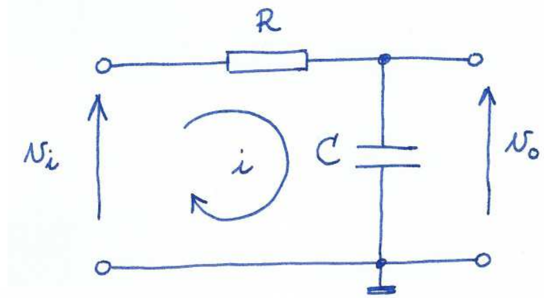
Kantelfrequentie f_k



Kantelfrequentie : f_k

$$\text{Stel } \left|\frac{v_o}{v_i}\right| = \frac{1}{\sqrt{2}} \approx 0{,}7 \quad (70\%)$$

Bepaal de frequentie waarbij 70% van de ingangsspanning (V_{in}) aan de uitgang (V_{out}) verschijnt.

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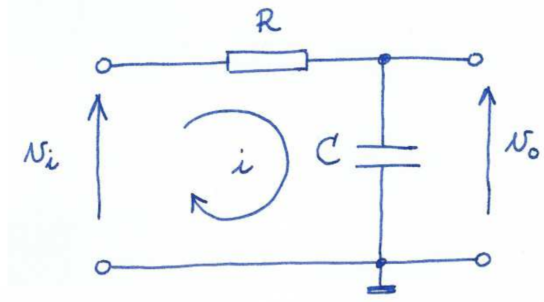
Kantelfrequentie f_k



Kantelfrequentie : f_k

Stel $\left|\frac{v_o}{v_i}\right| = \frac{1}{\sqrt{2}} \approx 0{,}7$ (70%)

$$\rightarrow \frac{1}{\sqrt{1+(\omega RC)^2}} = \frac{1}{\sqrt{2}}$$

A B

Deze uitdrukking is enkel waar als A = B!

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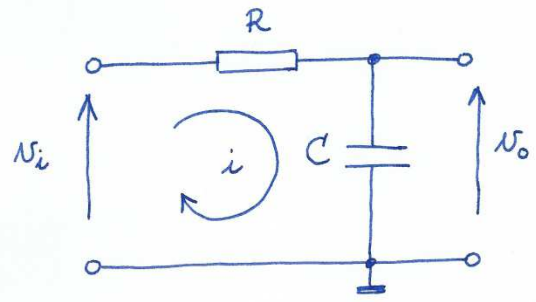
Kantelfrequentie f_k



$$\rightarrow \frac{1}{\sqrt{1+(\omega RC)^2}} = \frac{1}{\sqrt{2}}$$

$$\Leftrightarrow \sqrt{1+(\omega RC)^2} = \sqrt{2}$$

Linkerlid en rechterlid kwadrateren!

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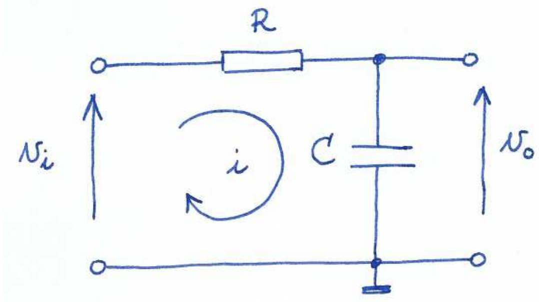
Kantelfrequentie f_k



$$\rightarrow \frac{1}{\sqrt{1+(\omega RC)^2}} = \frac{1}{\sqrt{2}}$$

$$\Leftrightarrow \sqrt{1+(\omega RC)^2} = \sqrt{2}$$

$$\Leftrightarrow 1+(\omega RC)^2 = 2$$

'1' overbrengen naar rechterlid

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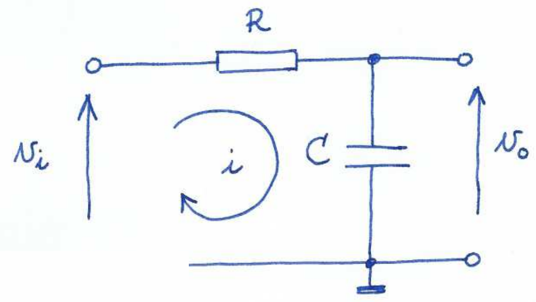
Kantelfrequentie f_k



$$\Leftrightarrow \sqrt{1+(\omega RC)^2} = \sqrt{2}$$

$$\Leftrightarrow 1+(\omega RC)^2 = 2$$

$$\Leftrightarrow (\omega RC)^2 = 2-1$$

$$\Leftrightarrow \omega^2(RC)^2 = 1$$

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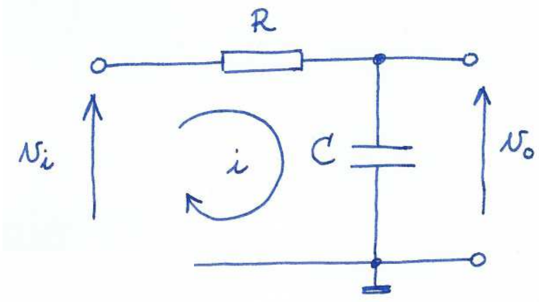
Kantelfrequentie f_k



$$\Leftrightarrow \sqrt{1+(\omega RC)^2} = \sqrt{2}$$

$$\Leftrightarrow 1+(\omega RC)^2 = 2$$

$$\Leftrightarrow (\omega RC)^2 = 2-1$$

$$\Leftrightarrow \omega^2 (RC)^2 = 1$$

Teller en noemer
delen door '(RC)²'

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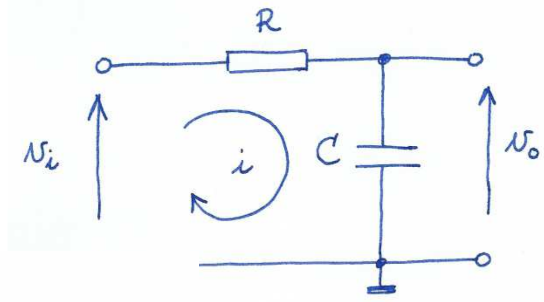
Kantelfrequentie f_k



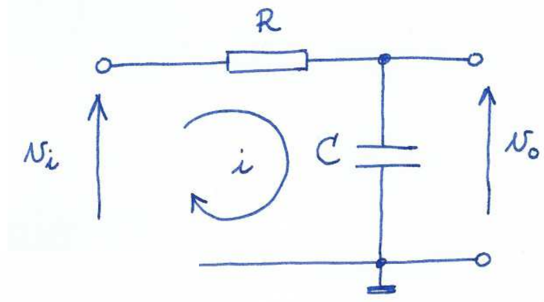
$$\Leftrightarrow \quad (\omega RC)^2 = 2-1$$

$$\Leftrightarrow \quad \omega^2 (RC)^2 = 1$$

$$\Leftrightarrow \quad \omega^2 = \frac{1}{(RC)^2}$$

RC-LAAGDOORLAATFILTER

Kantelfrequentie f_k



$$\Leftrightarrow \quad (\omega RC)^2 = 2-1$$

$$\Leftrightarrow \quad \omega^2 (RC)^2 = 1$$

$$\Leftrightarrow \quad \omega^2 = \frac{1}{(RC)^2}$$

Vierkantswortel van linkerlid en rechterlid

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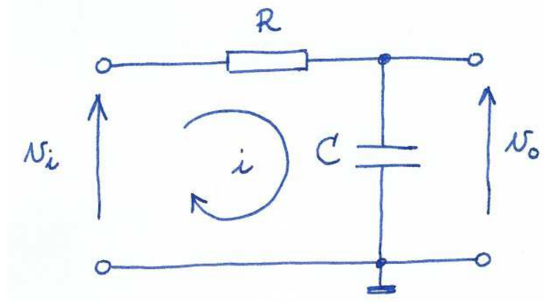
Kantelfrequentie f_k



$$\Leftrightarrow \quad (\omega RC)^2 = 2 - 1$$

$$\Leftrightarrow \quad \omega^2 (RC)^2 = 1$$

$$\Leftrightarrow \quad \omega^2 = \frac{1}{(RC)^2}$$

$$\Leftrightarrow \quad \omega = \sqrt{\frac{1}{(RC)^2}} = \frac{1}{RC}$$

Vierkantswortel van teller en noemer

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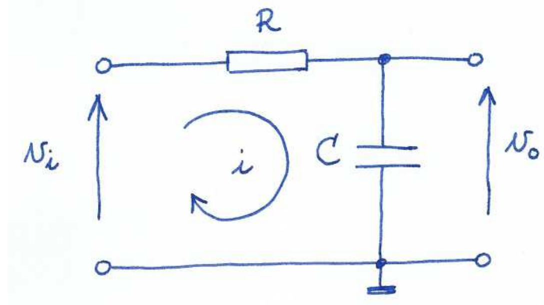
Kantelfrequentie f_k



$$\Leftrightarrow \quad \omega = \sqrt{\frac{1}{(RC)^2}} = \frac{1}{RC}$$

$$\Rightarrow \quad 2\pi f_k = \frac{1}{RC}$$

$$\Leftrightarrow \quad f_k = \frac{1}{2\pi RC}$$

$\omega = 2 . \pi . f$

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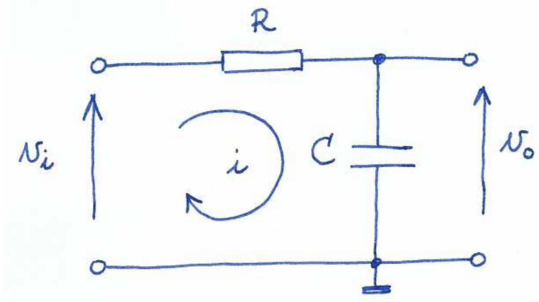
Kantelfrequentie f_k



$$\Leftrightarrow \quad \omega = \sqrt{\frac{1}{(RC)^2}} = \frac{1}{RC}$$

$$\Rightarrow \quad 2\pi f_k = \frac{1}{RC}$$

$$\Leftrightarrow \quad \boxed{f_k = \frac{1}{2\pi RC}}$$

Finale uitdrukking
voor de kantelfrequentie f_k

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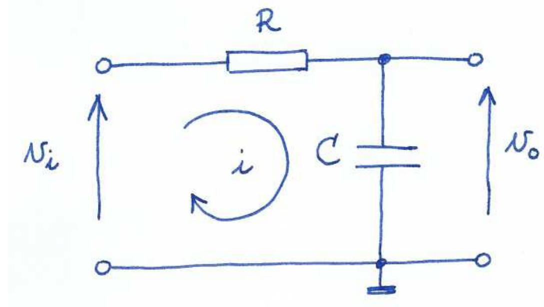
Tijdconstante τ



$$\Rightarrow \quad 2\pi f_k = \frac{1}{RC}$$

$$\Leftrightarrow \quad f_k = \frac{1}{2\pi RC}$$

Stel R . C = τ
(tijdconstante)

Alternatieve notatie:

$$f_k = \frac{1}{2 \,.\, \pi \,.\, \tau}$$

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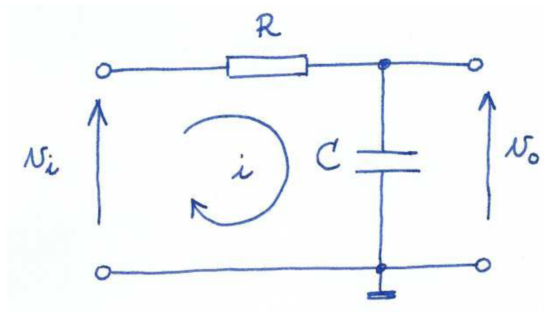
Frequentie-onderzoek



Voor een set van frequenties bepalen we:

- de spanningsversterking: A_v
- de fasehoek tussen IN en OUT: φ
- Dit resulteert in een AMPLITUDEKARAKTERISTIEK en een FASEKARAKTERISTIEK

HOE?

Hiervoor gaan we gebruik maken van een rekenblad (Excel).

Onderwerp voor een volgende tutorial!

ELECTRIVIAL TUTORIALS

EINDE

CREATIE & VOICE-OVER

W. Van Wichelen

GEBRUIKTE SOFTWARE

iSpring Free Cam

DATUM SCREENCAST

2021.01.04

DATUM PUBLICATIE

2021.01.04

DOELPUBLIEK

Industriële ICT

LEERPLANDOELEN

OO-2017-005/41/42/46

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